Pure Mathematics 4 formulae!

Proof:

- by contradiction ① assume statement (negation of statement) is not true
  ② work your way to contradiction of your statement or a known fact
  ③ mention your statement is false & the original one is true

- rational number ⇒ $\frac{a}{b}$ with no common factors
- irrational number ⇒ $\frac{a\sqrt{k}}{b}$ - where k is not square number

Partial fractions:

- $\frac{5}{(x+2)(x-4)} = \frac{A}{(x+2)} + \frac{B}{(x-4)}$ $\quad \therefore 5 = A(x-4) + B(x+2)$
- $\frac{5}{(x-2)^2(x+1)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x+1)}$
  └ repeated factor / root
- improper fraction is top heavy (power of polynomial in numerator ≥ that in denominator)

→ $\frac{x^2}{(x+1)(x-1)} = A + \frac{B}{x+1} + \frac{C}{x-1}$

→ $\frac{x^3}{(x+1)(x-1)} = Ax + B + \frac{C}{x+1} + \frac{D}{x-1}$

→ $\frac{2x}{(x+1)} = A + \frac{B}{x+1}$

→ $\frac{3x^3}{(x+1)^3} = A + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3}$

- improper → mixed
a) algebraic division
b) $f(x) = Q(x) \cdot \text{divisor} + \text{remainder}$
$\frac{f(x)}{\text{divisor}} = Q(x) + \frac{\text{remainder}}{\text{divisor}}$

(- Rasha's Notes 2023
hope they are of use!
Best of luck, you've got it!!)

Coordinate geometry in the (x-y) plane

- parametric ⟶ cartesian (subsitution to eliminate parameter)
- domain of $f(x)$ ⟶ range of $x = p(t)$
- range of $f(x)$ ⟶ range of $y = q(t)$
- all angles are in radians

Binomial expansion

- for $n \Rightarrow$ natural no.

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 \dots + b^n$$

- for $n \Rightarrow$ negative no. / fraction

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{6!}x^3 \dots$$

- ↳ valid for $|x|<1$ ⟶ forms an infinite series
  ↳ $-1<x<1$
- validity for $(1+bx)^n$ ⟶ $|bx|<1$
  $|b|\cdot|x|<1$
  $|x|<\frac{1}{|b|}$
  - ↳ high powers become insignificant for small values of x
- $(a+bx)^n \Rightarrow a^n\left(1+\frac{b}{a}x\right)^n$ ⟶ $\left|\frac{bx}{a}\right|<1$
  $|x|<\left|\frac{a}{b}\right|$
- if possible always use partial fractions

Differentiation

- if x & y are given as functions of parameter t: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$
- $\frac{d(f(y))}{dx} = f'(y)\cdot\frac{dy}{dx}$
- $\frac{d\,y^n}{dx} = ny^{n-1}\cdot\frac{dy}{dx}$
- $\frac{d\,xy}{dx} = y + x\frac{dy}{dx}$ $\quad (xy)'$ $\quad y + \frac{dy}{dx}x$
- $\frac{dy}{dx} = 0$ then $dy = 0$ ⟶ $\frac{dy}{dt} = 0$ ∴ $y = q(t)$ stationary point
- chain rule is holy tool for rate of Δ

integration

- $x = f(t)$
  $y = g(t)$
  area $= \int_{x1}^{x2} y\,dx = \int_{t1}^{t2} y \cdot dt \cdot f'(t)$

- volume $= \pi \int_a^b y^2\,dx$ (rotated 2π about x axis)
  $= \pi \int_{t1}^{t2} y^2 \cdot f'(t)\,dt$

- for substitution choose simplest one (pay attention to limits, in terms of u)

- by parts $\Rightarrow \int y \frac{dv}{dx}\,dx = u \cdot v - \int v \cdot \frac{du}{dx} \cdot dx$
  - pls pay attention to partial fractions they are everywhere

- integrating differential equations:
  $\frac{dy}{dx} = f(x)\,g(y)$ $\int \frac{dy}{g(y)} = \int f(x)\,dx$
  - general → c
  - particular → find c
    - use boundary condition ($x=1$ when $y=4$)

vectors

- column vector $\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix}$
- unit vectors: $i \Rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $j \Rightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $k \Rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$
  - $\hat{a}$ = unit vector $\frac{a}{|a|}$ (vector/magnitude)
- $a = xi + yj + zk$
- angle made with:
  - positive x axis → $\cos\theta_x = \frac{x}{|a|}$
  - positive y axis → $\cos\theta_y = \frac{y}{|a|}$
  - positive z axis → $\cos\theta_z = \frac{z}{|a|}$

→ angle between line & set plane is angle between line & its projection in that plane
- $\cos\theta = \frac{|\text{projection}|}{|\text{vector}|}$ → same as $\cos\theta_x = \frac{x}{|\text{vector}|}$
  - projection → wanted (projection plane)
  - vector as whole

- to proof point is midpoint (p) find two forms of $\overrightarrow{OP}$ using two constants & then equate, $p = \lambda = \gamma_2$
- $(x, y, z) \longrightarrow$ mag $\sqrt{x^2+y^2+z^2}$
- $r = a + \lambda(b)$
  - a direction vector
- dot product $\longrightarrow$ $a \cdot b = |a||b| \cos\theta$

  $x_1x_2 + y_1y_2 + z_1z_2 = |a||b|\cos\theta$

  (if parallel $\Rightarrow$ 1
  perpendicular $\Rightarrow$ 0

lines $\longrightarrow$ parallel (don't)
perpendicular (intersect)
skew (don't)

extra

- % error $= \frac{|T-E|}{T} \times 100$
- rate of change $\approx$ with respect to time
- tangent parrallel to x axis $\rightarrow \frac{dy}{dx} = 0$
  - yaxis $\longrightarrow \frac{dx}{dy} = 0$

- $\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$ $\quad$ $\int (ax+b)^n dx = (ax+b)^{n+1} \cdot \frac{1}{a} \cdot \frac{1}{n+1}$
- $\int e^{ax+b} dx = \frac{e^{ax+b}}{a} + c$
- $\sqrt{a^2-x^2}$ use $x = a\sin\theta$
- $\sqrt{a^2+x^2}$ use $x = a\tan\theta$
- $\sqrt{x^2-a^2}$ use $x = a\sec\theta$ or $\operatorname{cosec}\theta$
- LATE (algebraic, trig, exp, log) $\longrightarrow$ differintiate first, integrate second

two chords bisect one ote & midpoint is parallel to centre

improper fraction is one whose numerator has degree greater than or equal to denominator.

to convert : a) algebraic long division
b) relationship → $f(x) = Q(x) \cdot \text{divisor} + \text{remainder}$

2. $|a|$ is non negative value
a) $f(x) \geq 0$ then $|f(x)| = f(x)$
$f(x) < 0$ then $|f(x)| = -f(x)$

to sketch

4. $|2x+3| = 7$
argument of modulus

Mapping

one-one	many-one	one-many
· has inverse	· has restricted inverse	· no inverse

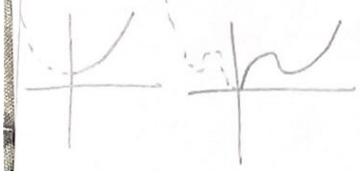
→ VLT → if it is function
HLT → type of function

inverses
· $f(x)$ & $f^{-1}(x)$ are inverses that is $ff^{-1}(x) = f^{-1}f(x) = x$
· graphs are reflections along line $y = x$
· domain of $f(x)$ is range of $f^{-1}(x)$ and vice versa
· on intersection solve $f(x) = x$

self inverse function is one where $f(x) = f^{-1}(x)$ here $ff(x) = x$

→ $10 - x$ let $x = 1$ ∴ $f(x) = 9$
so $ff(1) = 10 - 9 = 1$

$y = |f(x)|$



$y = |f(x)|$ reflected

· y axis reflection

$y = 3|x+1| - 2$ → vertex = $(-1, -2)$

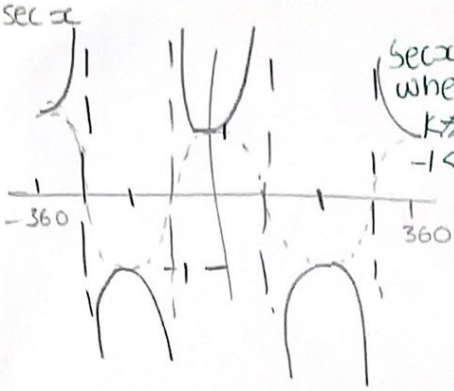
3. $\sec x = \frac{1}{\cos x}$

$\operatorname{cosec} x = \frac{1}{\sin x}$

$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

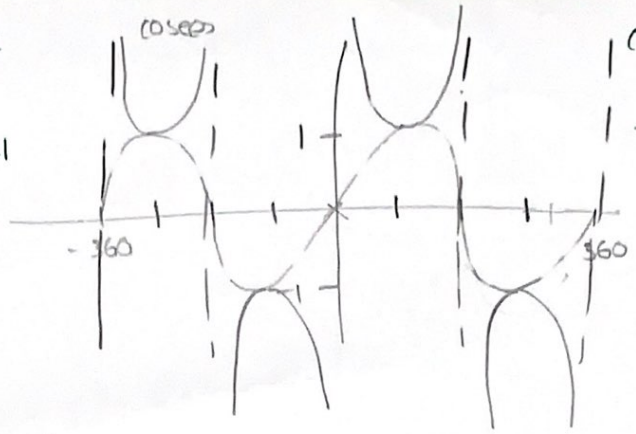
$x = 10 - y$
$x - 10 = -y$
$y = -x + 10$

sec x



sec x = k where $k \geq$ / $k \leq$ $-1 < k < 1$

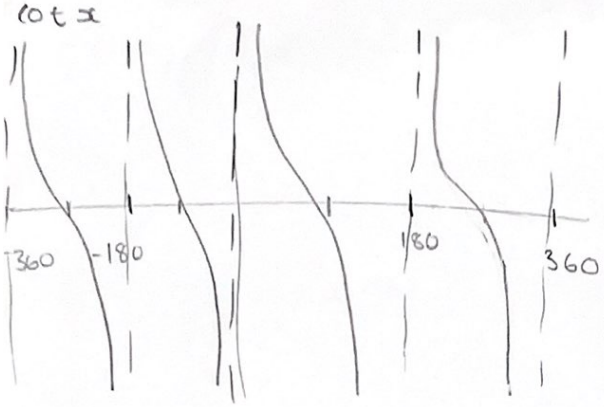
cosec



cosec x = k

$k \geq$ / $k \leq$ $-1 < k < 1$

cot x



$\cot = \frac{\cos}{\sin}$

can have solutions for any value of k

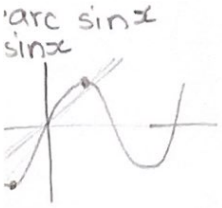
- $1 + \tan^2 x = \sec^2 x$

  $1 + \cot^2 x = \operatorname{cosec}^2 x$

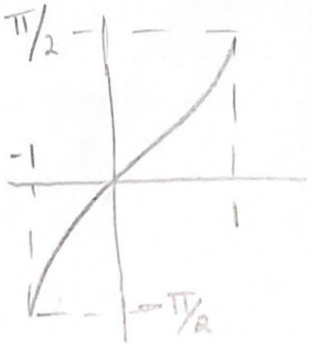
inverses

arc sin x

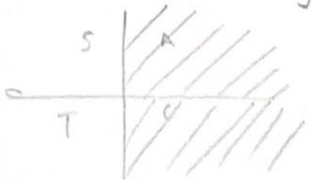
sin x



$\Rightarrow$

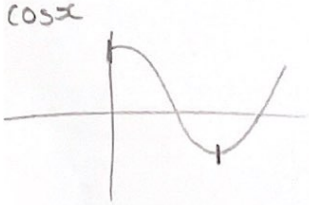


∴ inverse is only in range

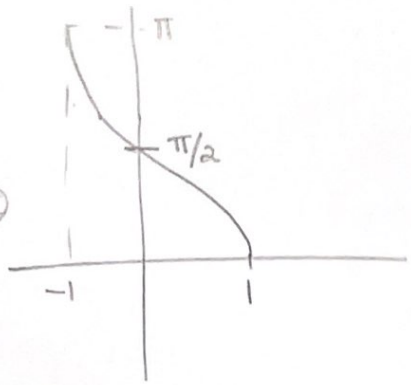


arc cos x

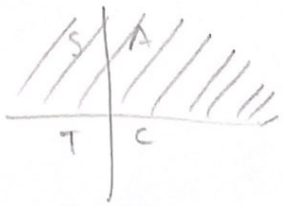
cos x



$\Rightarrow$



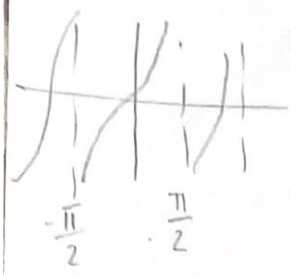
∴ inverse is only in



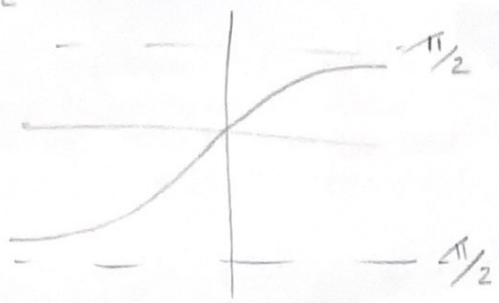
$\tan^{-1}x$

$\tan x$

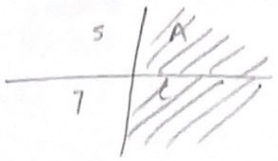
$\tan = \frac{s}{c}$



$\Rightarrow$



∴ inverse in range only



4.

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin(2A) = 2\sin A \cos A$

$\cos(2A) = \cos^2 A - \sin^2 A$

$= 2\cos^2 A - 1$

$= 1 - 2\sin^2 A$

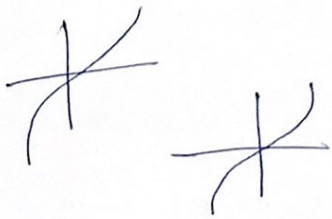
$\tan(2A) = \dfrac{2\tan A}{1 - \tan^2 A}$

* $\cos(-B) = +\cos B$

$\sin(-B) = -\sin B$

$\sin(x - \pi/2) = \cos x$

$\cos(x - \pi/2) = \sin x$



5.

$f(x) = e^x \therefore f'(x) = e^x$

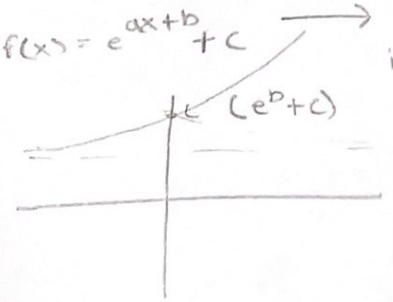
$f(x) = e^{kx} \therefore f'(x) = ke^{kx}$

$f(x) = e^{ax+b} + c \longrightarrow g(x) = e^x \rightarrow g(ax+b) + c$

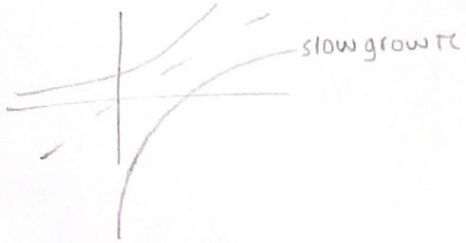
if $a > 1$

$a^{ax+b} + c$

$e^b + c$



$f^{-1}(e^x) = \ln(x)$



* $e^{\ln x} = y$

$\ln e^{\ln x} = \ln y$

$\ln x \cdot \ln e = \ln y$

$\ln y = \ln x$

$y = x$

(1) $y = ax^n$

$\log y = \log a + n \log x$

$y = c + mx$ $\quad \therefore \log y$ against $\log x$

$\therefore n =$ gradient

$c =$ intercept

(2) $y = ab^x$

$\log y = \log a + x \log b$

$y = c + xm$ $\quad \therefore \log y$ against x

$\therefore m = \log b =$ gradient

$c = \log a =$ intercept

6.

for small angles $\rightarrow \sin x \approx x$

$\cos x \approx 1 - \frac{1}{2}x^2$

$1 - \frac{1}{2}x^2$

$\therefore \lim_{n \to 0} \frac{\sin n}{n} = \lim_{n \to 0} \frac{n}{n} = 1$

$\lim_{n \to 0} \frac{\cos h - 1}{n} = \lim_{n \to 0} \frac{1 - \frac{1}{2}h^2 - 1}{n} = -\frac{1}{2}h = 0$

(4) $\frac{dy}{dx} \sin x \rightarrow \lim_{n \to 0} \frac{\sin(x+h) - \sin x}{h}$

$= \lim_{n \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{n}$

$= \lim_{h \to 0} \frac{\cos h \sin x}{n} + \frac{\sin h \cos x}{n} - \frac{\sin x}{n}$

$\frac{\sin x(\cos h)}{h} + \frac{\sin h \cos x}{n} - \frac{\sin x}{h}$

$= \lim_{n \to 0} \sin x \left(\frac{\cos h - 1}{h}\right) + \frac{\sin h}{n} \cos x$

$\sin x \left(\frac{\cos h h}{h} - \frac{1}{h}\right) + \cos x \rightarrow \sin x \left(\frac{-\frac{1}{2}h}{2}\right) + \cos x$

$= \lim_{n \to 0} \sin x \left(\frac{-\frac{1}{2}h}{2}\right) + 1 \cos x$

$\therefore \frac{dy}{dx} = \cos x$

* $f(x) = \sin kx \therefore \frac{dy}{dx} = k \cos kx$

$\frac{dy}{dx} \cos x \rightarrow \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$

$= \lim_{n \to 0} \frac{\cos x \cos h}{n} - \frac{\sin x \sin h}{n} - \frac{\cos x}{n}$

$= \lim_{h \to 0} \cos x \left(\frac{\cos h - 1}{h}\right) - \frac{\sin h}{n} \sin x$

$= \lim_{n \to 0} \cos x \underbrace{\left(\frac{1}{2}h\right)}_{0} - 1 \sin x$

$= \lim_{n \to 0} 0 \cos x - 1 \sin x \therefore \frac{dy}{dx} = -\sin x$

* $y = k \cos kx \therefore \frac{dy}{dx} = -k \sin kx$